Functions

CHAPTER 1

Concepts and Skills
In this chapter you will review precalculus topics. Although these topics are not directly tested on the AP exam, reviewing them will reinforce some basic principles:

- general properties of functions: domain, range, composition, inverse;
- special functions: absolute value, greatest integer; polynomial, rational, trigonometric, exponential, and logarithmic;

and the BC topic,

- parametrically defined curves

A. DEFINITIONS

Function
Domain
Range

A1. A *function f* is a correspondence that associates with each element a of a set called the *domain* one and only one element b of a set called the *range.* We write

$$f(a) = b$$

to indicate that b is the *value* of f at a. The elements in the domain are called *inputs*, and those in the range are called *outputs.*

A function is often represented by an equation, a graph, or a table.

A vertical line cuts the graph of a function in at most one point.

EXAMPLE 1

The domain of $f(x) = x^2 - 2$ is the set of all real numbers; its range is the set of all reals greater than or equal to -2. Note that

$$f(0) = 0^2 - 2 = -2, \quad f(-1) = (-1)^2 - 2 = -1,$$
$$f(\sqrt{3}) = (\sqrt{3})^2 - 2 = 1, \quad f(c) = c^2 - 2,$$
$$f(x + h) - f(x) = [(x + h)^2 - 2] - [x^2 - 2]$$
$$= x^2 + 2hx + h^2 - 2 - x^2 + 2 = 2hx + h^2.$$

EXAMPLE 2

Find the domains of: (a) $f(x) = \frac{4}{x-1}$; (b) $g(x) = \frac{x}{x^2-9}$; (c) $h(x) = \frac{\sqrt{4-x}}{x}$.

SOLUTIONS:

(a) The domain of $f(x) = \frac{4}{x-1}$ is the set of all reals except $x = 1$ (which we shorten to "$x \neq 1$").

(b) The domain of $g(x) = \frac{x}{x^2-9}$ is $x \neq 3, -3$.

(c) The domain of $h(x) = \frac{\sqrt{4-x}}{x}$ is $x \leqq 4, x \neq 0$ (which is a short way of writing $\{x \mid x \text{ is real}, x < 0 \text{ or } 0 < x \leqq 4\}$).

A2. Two functions f and g with the same domain may be combined to yield their sum and difference: $f(x) + g(x)$ and $f(x) - g(x)$, also written as $(f + g)(x)$ and $(f - g)(x)$, respectively; or their product and quotient: $f(x)g(x)$ and $f(x)/g(x)$, also written as $(fg)(x)$ and $(f/g)(x)$, respectively. The quotient is defined for all x in the shared domain except those values for which $g(x)$, the denominator, equals zero.

EXAMPLE 3

If $f(x) = x^2 - 4x$ and $g(x) = x + 1$, then find $\dfrac{f(x)}{g(x)}$ and $\dfrac{g(x)}{f(x)}$.

SOLUTIONS: $\dfrac{f(x)}{g(x)} = \dfrac{x^2 - 4x}{x+1}$ and has domain $x \neq -1$;

$$\frac{g(x)}{f(x)} = \frac{x+1}{x^2-4x} = \frac{x+1}{x(x-4)}$$ and has domain $x \neq 0, 4$.

Composition

A3. The *composition* (or *composite*) of f with g, written as $f(g(x))$ and read as "f of g of x," is the function obtained by replacing x wherever it occurs in $f(x)$ by $g(x)$. We also write $(f \circ g)(x)$ for $f(g(x))$. The domain of $(f \circ g)(x)$ is the set of all x in the domain of g for which $g(x)$ is in the domain of f.

EXAMPLE 4A

If $f(x) = 2x - 1$ and $g(x) = x^2$, then does $f(g(x)) = g(f(x))$?

SOLUTION: $f(g(x)) = 2(x^2) - 1 = 2x^2 - 1$

$g(f(x)) = (2x - 1)^2 = 4x^2 - 4x + 1.$

In general, $f(g(x)) \neq g(f(x))$.

EXAMPLE 4B

If $f(x) = 4x^2 - 1$ and $g(x) = \sqrt{x}$, find $f(g(x))$ and $g(f(x))$.

SOLUTIONS: $f(g(x)) = 4x - 1 \quad (x \geq 0)$; $g(f(x)) = \sqrt{4x^2 - 1} \quad (|x| \geq \frac{1}{2})$.

Symmetry

A4. A function f is $\begin{matrix} \textit{odd} \\ \textit{even} \end{matrix}$ if, for all x in the domain of f, $\begin{matrix} f(-x) = -f(x) \\ f(-x) = f(x) \end{matrix}$.

The graph of an odd function is symmetric about the origin; the graph of an even function is symmetric about the y-axis.

EXAMPLE 5

The graphs of $f(x) = \frac{1}{2}x^3$ and $g(x) = 3x^2 - 1$ are shown in Figure N1–1; $f(x)$ is odd, $g(x)$ even.

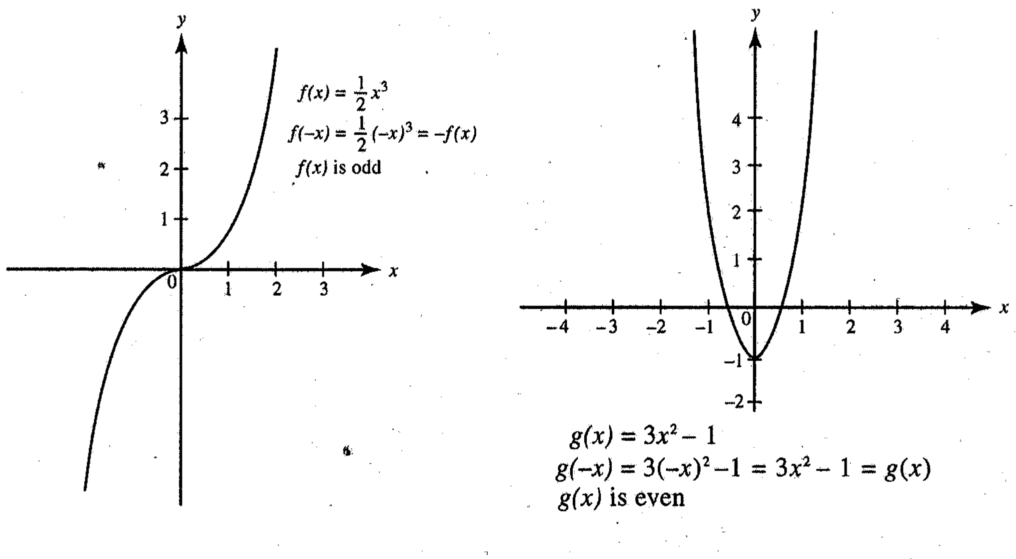


FIGURE N1–1

A5. If a function f yields a single output for each input and also yields a single input for every output, then f is said to be *one-to-one*. Geometrically, this means that any horizontal line cuts the graph of f in at most one point. The function sketched at the left in Figure N1–1 is one-to-one; the function sketched at the right is not. A function that is increasing (or decreasing) on an interval I is one-to-one on that interval (see pages 162–163 for definitions of increasing and decreasing functions).

A6. If f is one-to-one with domain X and range Y, then there is a function f^{-1}, with domain Y and range X, such that

$$f^{-1}(y_0) = x_0 \quad \text{if and only if} \quad f(x_0) = y_0.$$

Inverse

The function f^{-1} is the *inverse* of f. It can be shown that f^{-1} is also one-to-one and that its inverse is f. The graphs of a function and its inverse are symmetric with respect to the line $y = x$.

> To find the inverse of $y = f(x)$,
> interchange x and y,
> then solve for y.

EXAMPLE 6

Find the inverse of the one-to-one function $f(x) = x^3 - 1$.

SOLUTION: Interchange x and y: $\quad x = y^3 - 1$

Solve for y: $\quad y = \sqrt[3]{x+1} = f^{-1}(x)$.

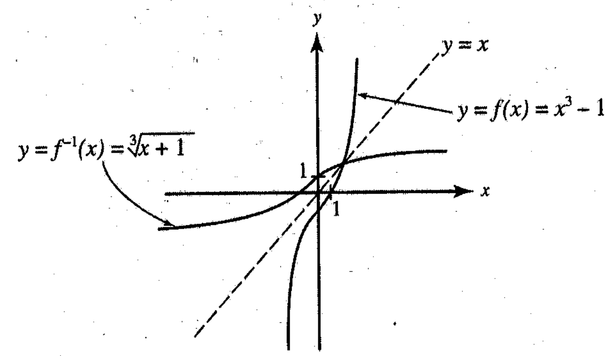


FIGURE N1–2

Note that the graphs of f and f^{-1} in Figure N1–2 are mirror images, with the line $y = x$ as the mirror.

Zeros

A7. The *zeros* of a function f are the values of x for which $f(x) = 0$; they are the x-intercepts of the graph of $y = f(x)$.

EXAMPLE 7

Find zeros of $f(x) = x^4 - 2x^2$.

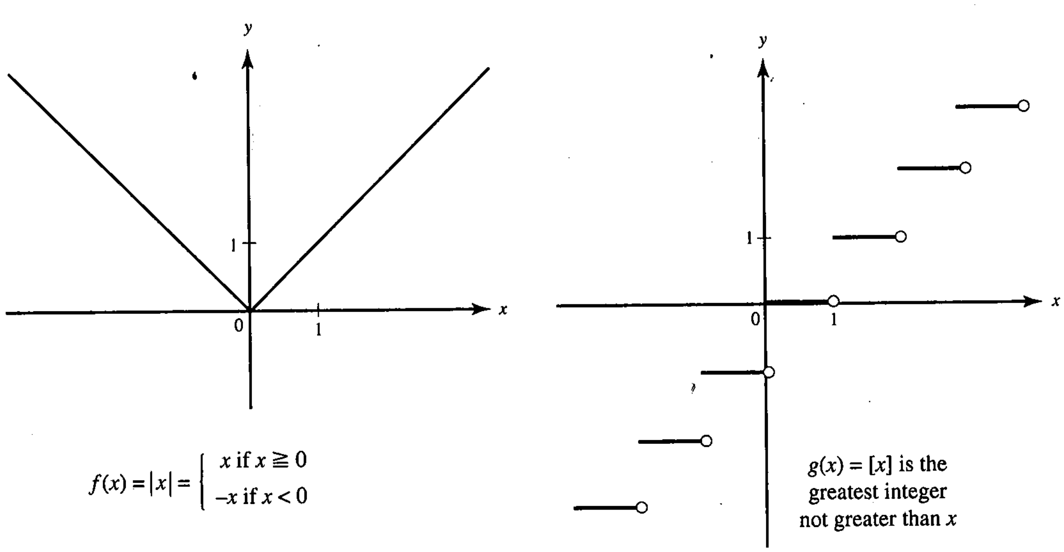
SOLUTION: The zeros are the x's for which $x^4 - 2x^2 = 0$. The function has three zeros, since $x^4 - 2x^2 = x^2(x^2 - 2)$ equals zero if $x = 0$, $+\sqrt{2}$, or $-\sqrt{2}$.

B. SPECIAL FUNCTIONS

The *absolute-value* function $f(x) = |x|$ and the *greatest-integer* function $g(x) = [x]$ are sketched in Figure N1–3.

Absolute value

Greatest integer



Absolute-value function

Greatest-integer function

FIGURE N1–3

EXAMPLE 8

A function f is defined on the interval $[-2, 2]$ and has the graph shown in Figure N1–4.

(a) Sketch the graph of $y = |f(x)|$.
(b) Sketch the graph of $y = f(|x|)$.
(c) Sketch the graph of $y = -f(x)$.
(d) Sketch the graph of $y = f(-x)$.

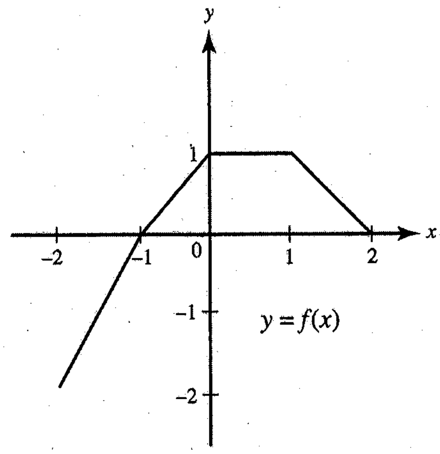


FIGURE N1–4

SOLUTIONS: The graphs are shown in Figures N1–4a through N1–4d.

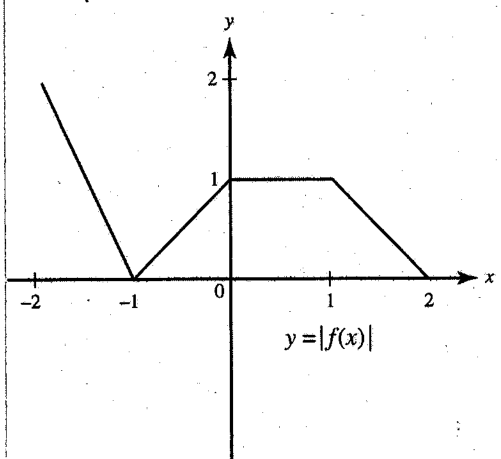


FIGURE N1–4a

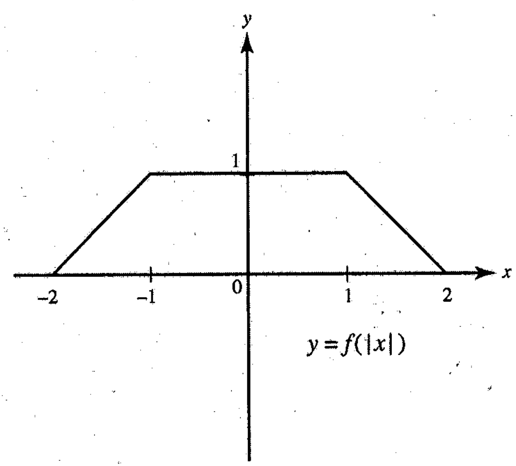


FIGURE N1–4b

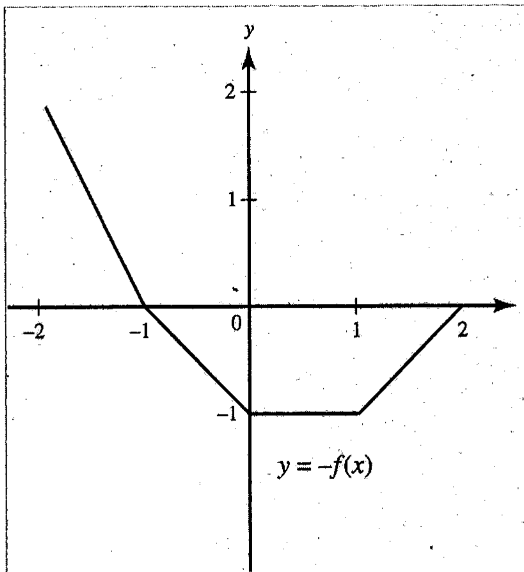


FIGURE N1–4c

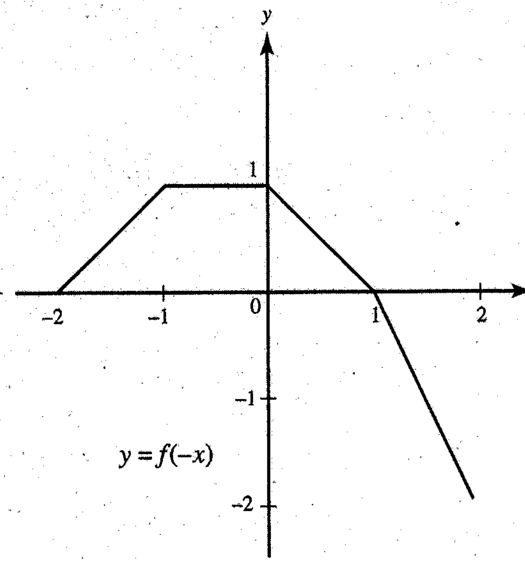


FIGURE N1–4d

Note that graph (c) of $y = -f(x)$ is the reflection of $y = f(x)$ in the x-axis, wheras Figure graph (d) of $y = f(-x)$ is the reflection of $y = f(x)$ in the y-axis. How do the graphs of $|f(x)|$ and $f(|x|)$ compare with the graph of $f(x)$?

EXAMPLE 9

Let $f(x) = x^3 - 3x^2 + 2$. Graph the following functions on your calculator in the window $[-3,3] \times [-3,3]$: (a) $y = f(x)$; (b) $y = |f(x)|$; (c) $y = f(|x|)$.

SOLUTIONS:

(a) $y = f(x)$
See Figure N1–5a.

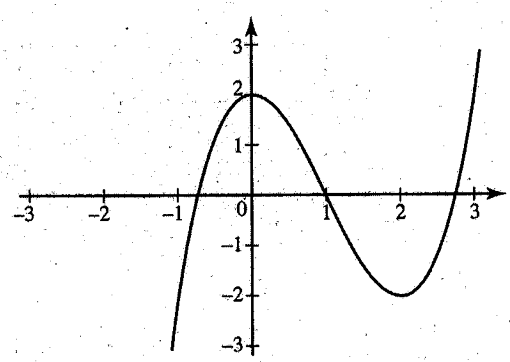


FIGURE N1–5a

(b) $y = |f(x)|$
See Figure N1–5b.

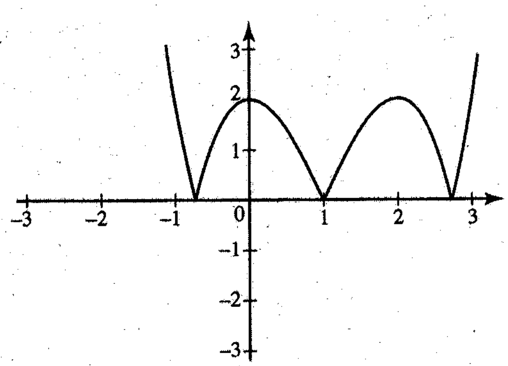


FIGURE N1–5b

(c) $y = f(|x|)$
See Figure N1–5c.

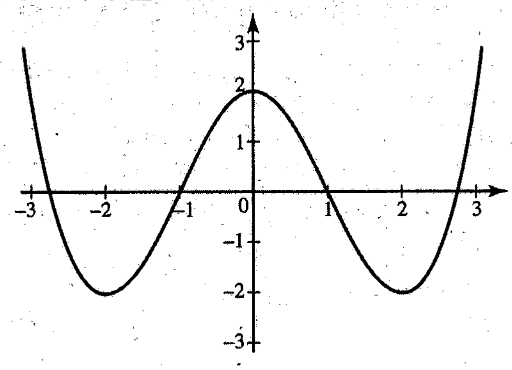


FIGURE N1–5c

Note how the graphs for (b) and (c) compare with the graph for (a).

C. POLYNOMIAL AND OTHER RATIONAL FUNCTIONS

C1. Polynomial Functions.

Polynomial

A *polynomial function* is of the form

$$f(x) = a_0x^n + a_1x^{n-1} + a_2x^{n-2} + \cdots + a_{n-1}x + a_n,$$

where n is a positive integer or zero, and the a_k's, the *coefficients,* are constants. If $a_0 \neq 0$, the degree of the polynomial is n.

A *linear* function, $f(x) = mx + b$, is of the first degree; its graph is a straight line with slope m, the constant rate of change of $f(x)$ (or y) with respect to x, and b is the line's y-intercept.

A *quadratic* function, $f(x) = ax^2 + bx + c$, has degree 2; its graph is a parabola that opens up if $a > 0$, down if $a < 0$, and whose axis is the line $x = -\frac{b}{2a}$.

A *cubic*, $f(x) = a_0x^3 + a_1x^2 + a_2x + a_3$, has degree 3; calculus enables us to sketch its graph easily; and so on. The domain of every polynomial is the set of all reals.

C2. Rational Functions.

Rational function

A *rational function* is of the form

$$f(x) = \frac{P(x)}{Q(x)},$$

where $P(x)$ and $Q(x)$ are polynomials. The domain of f is the set of all reals for which $Q(x) \neq 0$.

D. TRIGONOMETRIC FUNCTIONS

The fundamental trigonometric identities, graphs, and reduction formulas are given in the Appendix.

D1. Periodicity and Amplitude.

Trigonometric functions

The trigonometric functions are periodic. A function f is *periodic* if there is a positive number p such that $f(x + p) = f(x)$ for each x in the domain of f. The smallest such p is called the *period* of f. The graph of f repeats every p units along the x-axis. The functions $\sin x$, $\cos x$, $\csc x$, and $\sec x$ have period 2π; $\tan x$ and $\cot x$ have period π.

The function $f(x) = A \sin bx$ has amplitude A and period $\frac{2\pi}{b}$; $g(x) = \tan cx$ has period $\frac{\pi}{c}$.

EXAMPLE 10

Consider the function $f(x) = \frac{1}{k} \cos (kx)$.
(a) For what value of k does f have period 2?
(b) What is the amplitude of f for this k?

SOLUTIONS:
(a) Function f has period $\frac{2\pi}{k}$; since this must equal 2, we solve the equation $\frac{2\pi}{k} = 2$, getting $k = \pi$.

(b) It follows that the amplitude of f that equals $\frac{1}{k}$ has a value of $\frac{1}{\pi}$.

EXAMPLE 11

Consider the function $f(x) = 3 - \sin \frac{\pi x}{3}$.
Find (a) the period and (b) the maximum value of f.
(c) What is the smallest positive x for which f is a maximum?
(d) Sketch the graph.

SOLUTIONS:
(a) The period of f is $2\pi \div \frac{\pi}{3}$, or 6.
(b) Since the maximum value of $-\sin x$ is $-(-1)$ or $+1$, the maximum value of f is $3 + 1$ or 4.
(c) $-\left(\sin \frac{\pi x}{3}\right)$ equals $+1$ when $\sin \frac{\pi x}{3} = -1$, that is, when $\frac{\pi x}{3} = \frac{3\pi}{2}$. Solving yields $x = \frac{9}{2}$.
(d) We graph $y = 3 - \sin \frac{\pi x}{3}$ in $[-5,8] \times [0,5]$:

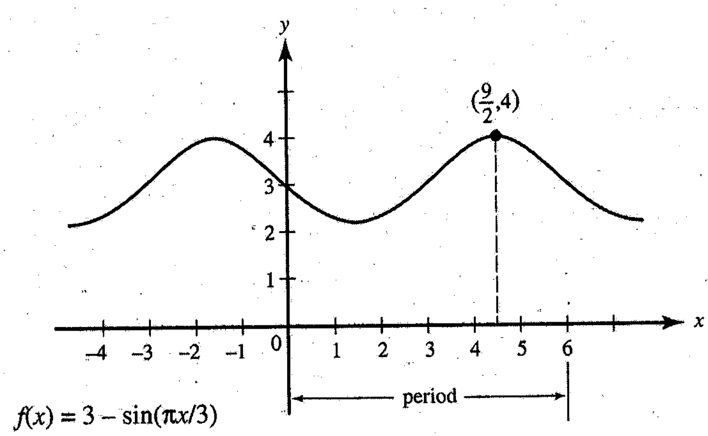


FIGURE N1–6

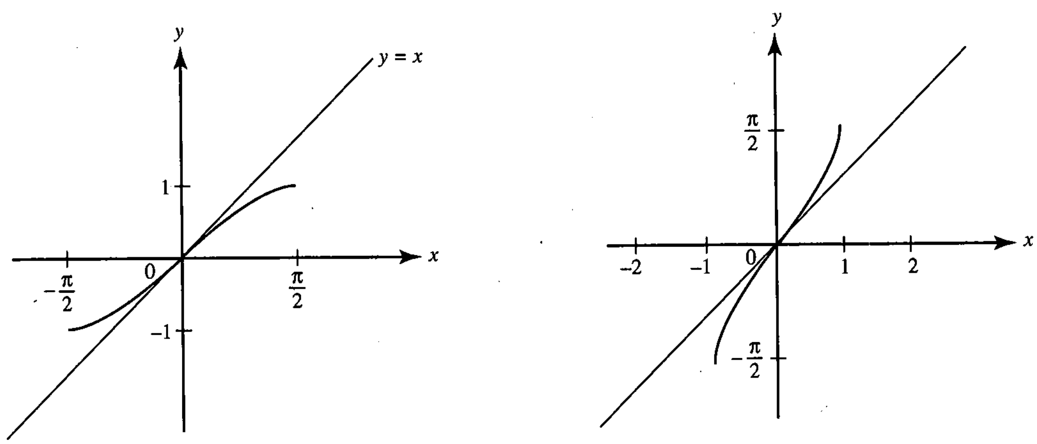
D2. Inverses.

Inverse trig functions

We obtain *inverses* of the trigonometric functions by limiting the domains of the latter so each trigonometric function is one-to-one over its restricted domain. For example, we restrict

$$\sin x \text{ to } -\frac{\pi}{2} \leqq x \leqq \frac{\pi}{2},$$
$$\cos x \text{ to } \quad 0 \leqq x \leqq \pi,$$
$$\tan x \text{ to } -\frac{\pi}{2} < x < \frac{\pi}{2}.$$

The graphs of $f(x) = \sin x$ on $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ and of its inverse $f^{-1}(x) = \sin^{-1}x$ are shown in Figure N1–7. The inverse trigonometric function $\sin^{-1}x$ is also commonly denoted by arcsin x, which denotes *the* angle whose sine is x. The graph of $\sin^{-1}x$ is, of course, the reflection of the graph of $\sin x$ in the line $y = x$.



$y = \sin x$
domain: $-\frac{\pi}{2} \leqq x \leqq \frac{\pi}{2}$
range: $-1 \leqq y \leqq 1$

$y = \sin^{-1}x = \arcsin x$
domain: $-1 \leqq x \leqq 1$
range: $-\frac{\pi}{2} \leqq y \leqq \frac{\pi}{2}$

FIGURE N1–7

Also, for other inverse trigonometric functions,

$y = \cos^{-1} x$ (or $\arccos x$) has domain $-1 \leqq x \leqq 1$ and range $0 \leqq y \leqq \pi$;

$y = \tan^{-1} x$ (or $\arctan x$) has domain the set of reals and range $-\dfrac{\pi}{2} < y < \dfrac{\pi}{2}$.

Note also that

$$\sec^{-1}(x) = \cos^{-1}\left(\frac{1}{x}\right), \quad \csc^{-1}(x) = \sin^{-1}\left(\frac{1}{x}\right), \quad \text{and} \quad \cot^{-1}(x) = \frac{\pi}{2} - \tan^{-1}(x).$$

E. EXPONENTIAL AND LOGARITHMIC FUNCTIONS

E1. Exponential Functions.

Exponential functions

The following laws of exponents hold for all rational m and n, provided that $a > 0$, $a \neq 1$:

$$a^0 = 1; \qquad a^1 = a; \qquad a^m \cdot a^n = a^{m+n}; \qquad a^m \div a^n = a^{m-n};$$

$$(a^m)^n = a^{mn}; \qquad a^{-m} = \frac{1}{a^m}.$$

The exponential function $f(x) = a^x$ ($a > 0$, $a \neq 1$) is thus defined for all real x; its domain is the set of positive reals. The graph of $y = a^x$, when $a = 2$, is shown in Figure N1–8.

Of special interest and importance in the calculus is the exponential function $f(x) = e^x$, where e is an irrational number whose decimal approximation to five decimal places is 2.71828. We define e on page 97.

E2. Logarithmic Functions.

Log functions

Since $f(x) = a^x$ is one-to-one, it has an inverse, $f^{-1}(x) = \log_a x$, called the *logarithmic* function with base a. We note that

$$y = \log_a x \quad \text{if and only if} \quad a^y = x.$$

The domain of $\log_a x$ is the set of positive reals; its range is the set of all reals. It follows that the graphs of the pair of mutually inverse functions $y = 2^x$ and $y = \log_2 x$ are symmetric to the line $y = x$, as can be seen in Figure N1–8.

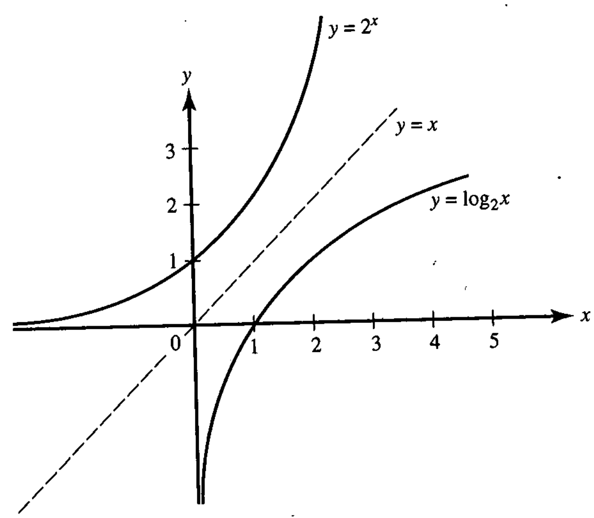


FIGURE N1–8

The logarithmic function $\log_a x$ ($a > 0$, $a \neq 1$) has the following properties:

$$\log_a 1 = 0; \qquad \log_a a = 1; \qquad \log_a mn = \log_a m + \log_a n;$$

$$\log_a \frac{m}{n} = \log_a m - \log_a n; \qquad \log_a x^m = m \log_a x.$$

The logarithmic base e is so important and convenient in calculus that we use a special symbol:

$$\log_e x = \ln x.$$

Logarithms with base e are called *natural* logarithms. The domain of $\ln x$ is the set of positive reals; its range is the set of all reals. The graphs of the mutually inverse functions $\ln x$ and e^x are given in the Appendix.

F. PARAMETRICALLY DEFINED FUNCTIONS

BC ONLY

If the x- and y-coordinates of a point on a graph are given as functions f and g of a third variable, say t, then

$$x = f(t), \qquad y = g(t)$$

Parametric equations

are called *parametric equations* and t is called the *parameter.* When t represents time, as it often does, then we can view the curve as that followed by a moving particle as the time varies.

EXAMPLE 12

Find the Cartesian equation of, and sketch, the curve defined by the parametric equations

$$x = 4 \sin t, \qquad y = 5 \cos t \qquad (0 \leqq t \leqq 2\pi).$$

SOLUTION: We can eliminate the parameter t as follows:

$$\sin t = \frac{x}{4}, \qquad \cos t = \frac{y}{5}.$$

Since $\sin^2 t + \cos^2 t = 1$, we have

$$\left(\frac{x}{4}\right)^2 + \left(\frac{y}{4}\right)^2 = 1 \text{ or } \frac{x^2}{16} + \frac{y^2}{25} = 1$$

The curve is the ellipse shown in Figure N1–9.

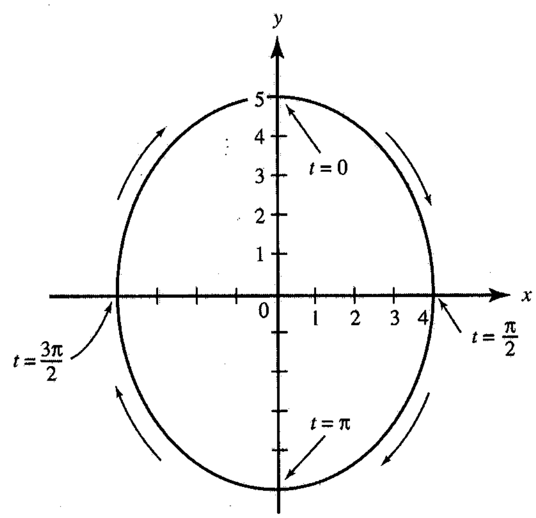


FIGURE N1–9

Note that, as t increases from 0 to 2π, a particle moving in accordance with the given parametric equations starts at point (0, 5) (when $t = 0$) and travels in a clockwise direction along the ellipse, returning to (0, 5) when $t = 2\pi$.

BC ONLY

EXAMPLE 13

Given the pair of parametric equations,

$$x = 1 - t, \quad y = \sqrt{t} \quad (t \geqq 0),$$

write an equation of the curve in terms of x and y, and sketch the graph.

SOLUTION: We can eliminate t by squaring the second equation and substituting for t in the first; then we have

$$y^2 = t \quad \text{and} \quad x = 1 - y^2.$$

We see the graph of the equation $x = 1 - y^2$ on the left in Figure N1–10. At the right we see only the upper part of this graph, the part defined by the parametric equations for which t and y are both restricted to nonnegative numbers.

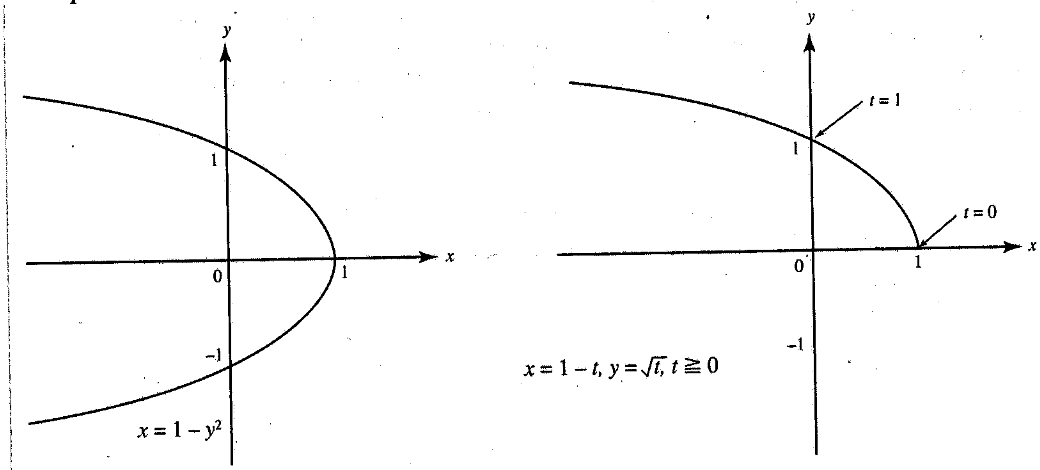


FIGURE N1–10

The function defined by the parametric equations here is $y = F(x) = \sqrt{1-x}$, whose graph is at the right above; its domain is $x \leqq 1$ and its range is the set of nonnegative reals.

EXAMPLE 14

A satellite is in orbit around a planet that is orbiting around a star. The satellite makes 12 orbits each year. Graph its path given by the parametric equations

$$x = 4\cos t + \cos 12t,$$
$$y = 4\sin t + \sin 12t.$$

SOLUTION: Shown below is the graph of the satellite's path using the calculator's parametric mode for $0 \leq t \leq 2\pi$.

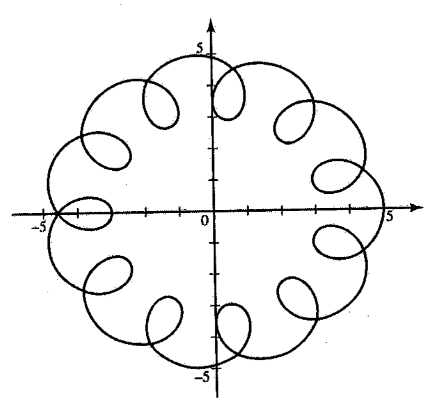


FIGURE N1–11

BC ONLY

EXAMPLE 15

Graph $x = y^2 - 6y + 8$.

SOLUTION: We encounter a difficulty here. The calculator is constructed to graph y as a function of x: it accomplishes this by scanning horizontally across the window and plotting points in varying vertical positions. Ideally, we want the calculator to scan *down* the window and plot points at appropriate horizontal positions. But it won't do that.

One alternative is to interchange variables, entering x as Y_1 and y as X, thus entering $Y_1 = X^2 - 6X + 8$. But then, during all subsequent processing we must remember that we have made this interchange.

Less risky and more satisfying is to switch to parametric mode: Enter $x = t^2 - 6t + 8$ and $y = t$. Then graph these equations in $[-10,10] \times [-10,10]$, for t in $[-10,10]$. See Figure N1–12.

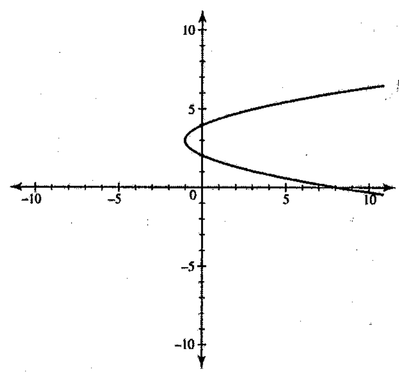


FIGURE N1–12

EXAMPLE 16

Let $f(x) = x^3 + x$; graph $f^{-1}(x)$.

SOLUTION: Recalling that f^{-1} interchanges x and y, we use parametric mode to graph

$$f:\ x = t,\ y = t^3 + t$$
$$\text{and } f^{-1}:\ x = t^3 + t,\ y = t.$$

Figure N1–13 shows both $f(x)$ and $f^{-1}(x)$.

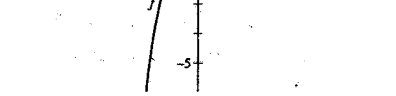


FIGURE N1–13

Parametric equations give rise to vector functions, which will be discussed in connection with motion along a curve in Chapter 4.

Chapter Summary

This chapter has reviewed some important precalculus topics. These topics are not directly tested on the AP exam; rather, they represent basic principles important in calculus. These include finding the domain, range and inverse of a function; and understanding the properties of polynomial and rational functions, trigonometric and inverse trig functions, and exponential and logarithmic functions.

For BC students, this chapter also reviewed parametrically defined functions.

Practice Exercises

Directions: Answer these questions *without* using your calculator.

1. If $f(x) = x^3 - 2x - 1$, then $f(-2) =$

(A) -17 **(B)** -13 **(C)** -5 **(D)** -1 **(E)** 3

2. The domain of $f(x) = \dfrac{x-1}{x^2+1}$ is

(A) all $x \neq 1$ **(B)** all $x \neq 1, -1$ **(C)** all $x \neq -1$
(D) $x \geqq 1$ **(E)** all reals

3. The domain of $g(x) = \dfrac{\sqrt{x-2}}{x^2-x}$ is

(A) all $x \neq 0, 1$ **(B)** $x \leqq 2, x \neq 0, 1$ **(C)** $x \leqq 2$
(D) $x \geqq 2$ **(E)** $x > 2$

4. If $f(x) = x^3 - 3x^2 - 2x + 5$ and $g(x) = 2$, then $g(f(x)) =$

(A) $2x^3 - 6x^2 - 2x + 10$ **(B)** $2x^2 - 6x + 1$ **(C)** -6
(D) -3 **(E)** 2

5. With the functions and choices as in Question 4, which choice is correct for $f(g(x))$?

6. If $f(x) = x^3 + Ax^2 + Bx - 3$ and if $f(1) = 4$ and $f(-1) = -6$, what is the value of $2A + B$?

(A) 12 **(B)** 8 **(C)** 0 **(D)** -2
(E) It cannot be determined from the given information.

7. Which of the following equations has a graph that is symmetric with respect to the origin?

(A) $y = \dfrac{x-1}{x}$ **(B)** $y = 2x^4 + 1$ **(C)** $y = x^3 + 2x$

(D) $y = x^3 + 2$ **(E)** $y = \dfrac{x}{x^3+1}$

8. Let g be a function defined for all reals. Which of the following conditions is not sufficient to guarantee that g has an inverse function?

(A) $g(x) = ax + b, a \neq 0.$ **(B)** g is strictly decreasing.
(C) g is symmetric to the origin. **(D)** g is strictly increasing.
(E) g is one-to-one.

9. Let $y = f(x) = \sin(\arctan x)$. Then the range of f is

(A) $\{y \mid 0 < y \leqq 1\}$ **(B)** $\{y \mid -1 < y < 1\}$ **(C)** $\{y \mid -1 \leqq y \leqq 1\}$

(D) $\left\{y \mid -\frac{\pi}{2} < y < \frac{\pi}{2}\right\}$ **(E)** $\left\{y \mid -\frac{\pi}{2} \leqq y \leqq \frac{\pi}{2}\right\}$

10. Let $g(x) = |\cos x - 1|$. The maximum value attained by g on the closed interval $[0, 2\pi]$ is for x equal to

(A) -1 **(B)** 0 **(C)** $\frac{\pi}{2}$ **(D)** 2 **(E)** π

11. Which of the following functions is not odd?

(A) $f(x) = \sin x$ **(B)** $f(x) = \sin 2x$ **(C)** $f(x) = x^3 + 1$

(D) $f(x) = \frac{x}{x^2+1}$ **(E)** $f(x) = \sqrt[3]{2x}$

12. The roots of the equation $f(x) = 0$ are 1 and -2. The roots of $f(2x) = 0$ are

(A) 1 and -2 **(B)** $\frac{1}{2}$ and -1 **(C)** $-\frac{1}{2}$ and 1

(D) 2 and -4 **(E)** -2 and 4

13. The set of zeros of $f(x) = x^3 + 4x^2 + 4x$ is

(A) $\{-2\}$ **(B)** $\{0, -2\}$ **(C)** $\{0, 2\}$ **(D)** $\{2\}$ **(E)** $\{2, -2\}$

14. The values of x for which the graphs of $y = x + 2$ and $y^2 = 4x$ intersect are

(A) -2 and 2 **(B)** -2 **(C)** 2 **(D)** 0 **(E)** none of these

15. The function whose graph is a reflection in the y-axis of the graph of $f(x) = 1 - 3^x$ is

(A) $g(x) = 1 - 3^{-x}$ **(B)** $g(x) = 1 + 3^x$ **(C)** $g(x) = 3^x - 1$
(D) $g(x) = \log_3(x - 1)$ **(E)** $g(x) = \log_3(1 - x)$

16. Let $f(x)$ have an inverse function $g(x)$. Then $f(g(x)) =$

(A) 1 **(B)** x **(C)** $\frac{1}{x}$ **(D)** $f(x) \cdot g(x)$ **(E)** none of these

17. The function $f(x) = 2x^3 + x - 5$ has exactly one real zero. It is between

(A) -2 and -1 **(B)** -1 and 0 **(C)** 0 and 1
(D) 1 and 2 **(E)** 2 and 3

18. The period of $f(x) = \sin \frac{2\pi}{3} x$ is

(A) $\frac{1}{3}$ (B) $\frac{2}{3}$ (C) $\frac{3}{2}$ (D) 3 (E) 6

19. The range of $y = f(x) = \ln(\cos x)$ is

(A) $\{y \mid -\infty < y \leqq 0\}$ (B) $\{y \mid 0 < y \leqq 1\}$ (C) $\{y \mid -1 < y < 1\}$
(D) $\left\{y \mid -\frac{\pi}{2} < y < \frac{\pi}{2}\right\}$ (E) $\{y \mid 0 \leqq y \leqq 1\}$

20. If $\log_b(3^b) = \frac{b}{2}$, then $b =$

(A) $\frac{1}{9}$ (B) $\frac{1}{3}$ (C) $\frac{1}{2}$ (D) 3 (E) 9

21. Let f^{-1} be the inverse function of $f(x) = x^3 + 2$. Then $f^{-1}(x) =$

(A) $\frac{1}{x^3 - 2}$ (B) $(x+2)^3$ (C) $(x-2)^3$
(D) $\sqrt[3]{x+2}$ (E) $\sqrt[3]{x-2}$

22. The set of x-intercepts of the graph of $f(x) = x^3 - 2x^2 - x + 2$ is

(A) $\{1\}$ (B) $\{-1, 1\}$ (C) $\{1, 2\}$
(D) $\{-1, 1, 2\}$ (E) $\{-1, -2, 2\}$

23. If the domain of f is restricted to the open interval $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$, then the range of $f(x) = e^{\tan x}$ is

(A) the set of all reals (B) the set of positive reals
(C) the set of nonnegative reals (D) $\{y \mid 0 < y \leqq 1\}$
(E) none of these

24. Which of the following is a reflection of the graph of $y = f(x)$ in the x-axis?

(A) $y = -f(x)$ (B) $y = f(-x)$ (C) $y = |f(x)|$
(D) $y = f(|x|)$ (E) $y = -f(-x)$

25. The smallest positive x for which the function $f(x) = \sin\left(\frac{x}{3}\right) - 1$ is a maximum is

(A) $\frac{\pi}{2}$ (B) π (C) $\frac{3\pi}{2}$ (D) 3π (E) 6π

26. $\tan\left(\arccos\left(-\frac{\sqrt{2}}{2}\right)\right) =$

(A) -1 (B) $-\frac{\sqrt{3}}{3}$ (C) $-\frac{1}{2}$ (D) $\frac{\sqrt{3}}{3}$ (E) 1

27. If $f^{-1}(x)$ is the inverse of $f(x) = 2e^{-x}$, then $f^{-1}(x) =$

(A) $\ln\left(\frac{2}{x}\right)$ (B) $\ln\left(\frac{x}{2}\right)$ (C) $\left(\frac{1}{2}\right)\ln x$
(D) $\sqrt{\ln x}$ (E) $\ln(2-x)$

28. Which of the following functions does not have an inverse function?

(A) $y = \sin x \left(-\frac{\pi}{2} \leqq x \leqq \frac{\pi}{2}\right)$ (B) $y = x^3 + 2$ (C) $y = \frac{x}{x^2+1}$
(D) $y = \frac{1}{2}e^x$ (E) $y = \ln(x-2)$ (where $x > 2$)

29. Suppose that $f(x) = \ln x$ for all positive x and $g(x) = 9 - x^2$ for all real x. The domain of $f(g(x))$ is

(A) $\{x \mid x \leqq 3\}$ (B) $\{x \mid |x| \leqq 3\}$ (C) $\{x \mid |x| > 3\}$
(D) $\{x \mid |x| < 3\}$ (E) $\{x \mid 0 < x < 3\}$

30. Suppose (as in Question 29) that $f(x) = \ln x$ for all positive x and $g(x) = 9 - x^2$ for all real x. The range of $y = f(g(x))$ is

(A) $\{y \mid y > 0\}$ (B) $\{y \mid 0 < y \leqq \ln 9\}$ (C) $\{y \mid y \leqq \ln 9\}$
(D) $\{y \mid y < 0\}$ (E) none of these

BC ONLY

31. The curve defined parametrically by $x(t) = t^2 + 3$ and $y(t) = t^2 + 4$ is part of a(n)

(A) line (B) circle (C) parabola
(D) ellipse (E) hyperbola

32. Which equation includes the curve defined parametrically by $x(t) = \cos^2(t)$ and $y(t) = 2\sin(t)$?

(A) $x^2 + y^2 = 4$ (B) $x^2 + y^2 = 1$ (C) $4x^2 + y^2 = 4$
(D) $4x + y^2 = 4$ (E) $x + 4y^2 = 1$

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